A Method of Genetic Algorithm Based Multiobjective Optimization via Cooperative Coevolution

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The paper deals with the identification of Pareto optimal solutions using GA based coevolution in the context of multiobjective optimization. Coevolution is a genetic process by which several species work with different types of individuals in parallel. The concept of cooperative coevolution is adopted to compensate for each of single objective optimal solutions during genetic evolution. The present study explores the GA based coevolution, and develops prescribed and adaptive scheduling schemes to reflect design characteristics among single objective optimization. In the paper, non-dominated Pareto optimal solutions are obtained by controlling scheduling schemes and comparing each of single objective optimal solutions. The proposed strategies are subsequently applied to a three-bar planar truss design and an energy preserving flywheel design to support proposed strategies.

Key Words : Multiobjective Optimization, Pareto Optimal, Genetic Algorithm, Coevolution, Penalty on Difference

1. Introduction

Genetic algorithm (GA) and its enhanced versions have received recent considerable attention in areas of engineering design and optimization (Lee, 1996 ; Le Riche et al., 1993 ; Windhorst et al., 2004). GA has been shown to be effective when the analysis model is inherently nonlinear and the design problem is represented by a mixture of continuous, integer and/or discrete design variables (Hajela et al., 1995 ; Saxena, 2005 ; Vigdergauz, 2001). GA has the higher probability of locating a global optimum without evaluating derivative based sensitivity information. GA works through the evolution of multiple designs under the implicit parallelism. Biologically inspired operations of crossover and mutation facilitate to produce competitive genes during genetic evolution. Such mechanisms in GA improve the current level of the system performance, and eventually allow to obtain the near-global optimum state of the system under the given parameter environments. The distinct features of genetic algorithm draw upon diversity, discovery and adaptation. In GA, the system is adapted toward the maximum performance by discovering the new competitive genes among the diverse individuals. Among its features, the diversity would be recognized in a case where the multiple local optima are necessary to search, especially in the context of multi-criterion and/or multi-objective optimization.

Multi-objective optimization methods have been widely studied in order to resolve the demand such that the practical engineering design problems often require a number of design objectives that are purposely conflicted among them. Pareto optimization can be stated as the problem of determining an optimal solution based on multiple, possibly competing criteria. The solution to a multi-objective problem is, as a rule, not a particular value, but a set of values of decision vari-

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ables such that, for each element in this set, none of the objective functions can be further increased without a decrease of some of the remaining objective functions, i.e., every such value of a decision variable is referred to as PARETO-OPTIMAL (Mason et al., 1998). Pareto optimal solutions have also been termed non-dominated. This name arises from the fact that no other solution is superior to them in all objectives. In other words, the non-dominated solutions are those solutions that cannot be improved in all objectives simultaneously. There have been a number of literatures associated with multi-objective optimization using evolutionary computing and genetic algorithms such as vector evaluated genetic algorithm (Schaffer, 1985), multiobjective genetic algorithm (Fonseca et al., 1993 ; Horn et al., 1991 ; Narayanan et al., 1999 ; Obayashi et al., 1997 ; Zitsler et al., 1998), coevolutionary computing (Lohn et al., 2002 ; Parmee et al., 1999) and immune network simulation (Yoo et al., 1999a ; 1999b), etc.

The paper discusses the adaptation of coevolution in the context of GA based multiobjective optimization. Coevolution is a genetic process by which several species work with different types of individuals in parallel. In general, there are various interactions in two or more species. These interactions depend on the influence of a species relevant to the other. The natural evolution is simply the adaptation of an individual within a species to the fixed environment. However, actual, realistic evolution is the process of the interactions in different species and/or changeable environment. That is, creatures in a nature coevolve through various species and environment. Genetic algorithm is a computational model that mimics the natural evolution, but is powerful only in limited areas of applications due to its simplicity. The successfulness of GA depends on what type of measure of fitness is selected and how much the fitness function is reflected to a real situation. Coevolutionary computation (CEC) might be one of solutions to overcome the limitation of fitness. In CEC, the fitness of a species is influence by another evolving species, that is, more than two species affect their fitness spontaneously (Fogel et al., 2003). Coevolution is classified into several ways ; one is the competitive coevolution over different species such as prey and predator, the other is the cooperative coevolution in which species evolve through compensation. There is another coevolution that is related between a host and a number of parasites.

Multiobjective optimization is a design method to find the optimum set of design variables that contribute to all the objective functions, which is analogous to CEC in a case where individuals of different species are cooperatively coevolved by reflecting the fitness of each species. The concept of CEC has been explored in the preliminary airframe design, where 'constraint range map' is introduced to locate multiobjective solutions gradually, starting from each of the premature single-objective designs (Parmee et al., 1999). Constraint range map is a scheduling of how design solutions of a single objective optimization are reflected to another during the coevolution process of multiobjective optimization. The present study explores the GA based coevolution, and develops prescribed and adaptive scheduling schemes in CEC. In the paper, non-dominated Pareto optimal solutions are obtained by controlling scheduling schemes and comparing each of single objective optimal solutions. The proposed strategies are subsequently applied to a three-bar planar truss design and an energy preserving flywheel design.

2. Coevolution

2.1 Proposed strategy

The section discusses the procedure of coevolution strategies in GA based multiobjective optimization. Suppose for simplicity there are two constrained objective functions, f_1 and f_2 in the minimization problem such that :

$$\begin{array}{lll} \text{Minimize} & f_1(x_i) & f_2(x_i) \\ \text{subject to} & g(x_i) \le 0 & g(x_i) \le 0 \\ & x_i^{lower} \le x_{ii}^{upper} & x_i^{lower} \le x_i \le x_i^{upper} \end{array} \quad (1)$$

At first, define randomly generated initial populations of P1 and P2 corresponding to f_1 and f_2, respectively. The subsequent coevolution process is explained as follows:

(1) Perform the genetic evolution for P1 and P2 in parallel. After a generation, each evolved population is rearranged with the order of fitness.

(2) Select the best individual (i.e., the best design variable vector, $x_i^{best(P_2)}$, $i=1,\cdots,N$, where, N is the number of design variables) at P2, and compute the difference between design variable values as follows:

$$D_i=\left|\frac{x_i^{(P_1)}-x_i^{best(P_2)}}{x_i^{upper}-x_i^{lower}}\right| \tag{2}$$

where, $x_i^{(P_1)}$ is i-th design variable value in an individual at P1, and x_i^{lower} and x_i^{upper} are lower and upper bounds on design variable, respectively. Now, consider 'penalty on difference' (POD) as shown in Figure 2. If the value of D_i is greater than POD based on the difference between $x_i^{(P_1)}$ and $x_i^{best(P_2)}$, a penalty is added to a corresponding individual's constrained objective function value at P1.

(3) Evaluate D_i for all design variables in an individual at P1.

(4) Repeat Step 2) and Step 3) for all individuals at P1.

(5) Likewise, based on the best individual at P1, repeat Steps 2) to 4) for all design variables in all individuals at P2.

(6) Rearrange P1 and P2 in the order of coevolved fitness, and remove the fourth quarter of total individuals from each population. It is also necessary to exclude such lowest individuals from the participation into the subsequent genetic evolution. Instead, generate a quarter of individuals at random.

(7) Using newly constructed population, go back to Step 1) until GA convergence.

Stepwise process in 2) to 4) is depicted in Figure 1. There are four different behaviors in POD scheduling whose initial value is 1.0 (i.e., 100% in terms of lower and upper bounds on design variable) at the beginning of the generation and final value is 0.1 (i.e., 10% difference between individuals at P1 and P2). It is noted that POD value is reduced over the generation as

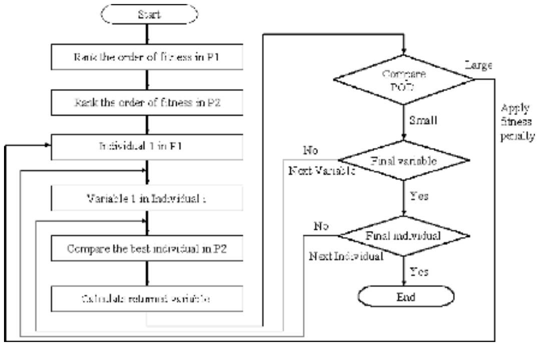


Fig. 1 Evaluation of POD in multiobjective optimization by GA based coevolution

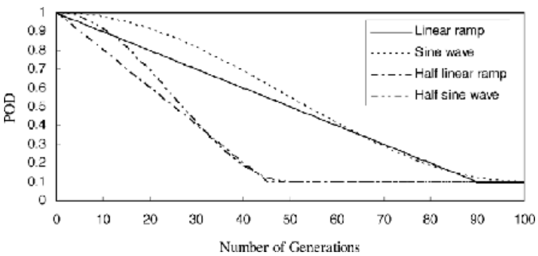


Fig. 2 Behaviors of prescribed POD's

shown in Figure 2. Such POD's are representatives in terms of nonlinearity and convergence speed; sine wave and half sine wave present nonlinearity in their behavior, and half linear and half sine wave show rapid convergence history. Since the optimal solution at P1 would not be obtained at the early stage of genetic evolution, D_i is allowed within a quite large value of POD. After a considerable number of generations, well-evolved designs at P1 should be much closer to the best design at P2 in the context of multiobjective design. By introducing D_i and POD, the characteristics of P2 can be reflected into the single objective optimization results at P1. In the proposed approach, the comparison between D_i and POD facilitates for P1 to conduct the single-objective optimization together with the reflection of the best design at P2. Through the above process, fully evolved individuals at P1 are consisted of designs whose objective function values are minimized (or reduced as much as possible), whose constraints are satisfied, and that are also similar to the best design at P2.

The condition of D_i being less than POD indicates the degree of how much designs in a certain population are similar to a design in another. The coevolution process employs four kinds of POD's in Figure 2, wherein the final value of POD is 0.1 as mentioned before. For example, the final value of $POD_{final}=1.0$ means that POD value is the same over the generation and there is no penalty for all designs in all individuals since D_i is always less than or equal to POD. In this case, the single objective optimization at P1 is separately conducted without including design characteristics of P2. Compared to a sense of weighted method in multiobjective optimization, $POD_{final}=1.0$ corresponds to the use of weighting factors, $\omega_1=1$ and $\omega_2=0$ when the weighting factor based multiobjective optimization is formulated as follows :

$$\text{Minimize} \quad \omega_1\frac{f_1(x_i)}{f_1^*}+\omega_2\frac{f_2(x_i)}{f_2^*}(\text{where, } \omega_1+\omega_2=1) \tag{3}$$

$$\text{subject to} \quad g(x_i)\leq 0, \quad x_i^{lower}\leq x_i\leq x_i^{upper}.$$

In Eq. (3), f_1^* are f_2^* are optimal objective function value obtained through single objective optimization. Likewise, when POD_{final} is 0.1 or smaller, coevolved designs could reflect the common designs each other, hence $POD_{final}=0.1$ roughly corresponds to $\omega_1=0.5$ and $\omega_2=0.5$. Now, non-dominated Pareto optimal solutions would be identified by changing the final value of POD ranging from between 0.0 and 1.0.

2.2 Adaptive POD

Aforementioned four different POD's in Figure 2 are basically a method of prescribed scheduling to account for inter-relations between P1 and P2. However, the POD value is different depending on design problems therefore it should be adaptively altered according to the result of coevolution process over the generation. In the adaptive scheduling, the current value of POD is determined based on how many designs or individuals are satisfied with the previous value of POD. The present study introduces a new scheme of 'adaptive penalty on difference' (APOD) as follows :

$$APOD^{(q)}=APOD^{(q-1)}\left[1-k\frac{NIND^{(q-1)}}{NPOP}\right] \tag{4}$$

where, $NIND$ is the number of individuals that satisfy the previous value of APOD. $NPOP$ is the number of individuals in a population, and q denotes a generation counter. A relaxation parameter, k ranging from 0.1 to 1.0 is used to control APOD value as well. This parameter reflects the change in APOD. Eq. (4) enables the current value of APOD to be tighter in a case where the number of individuals that satisfy the previous value of APOD is increased, thus the speed-up of coevolution process would be available. In the use of APOD, Pareto optimal solutions are obtained by terminating the coevolution process when such APOD is reached at a specified value, e.g., 0.9, 0.8 or 0.1.

3. Design Problems

3.1 Three-bar truss

The design objective is to determine the optimal cross sectional areas, A_1 and A_2 by minimizing both the total weight (W) of a statically loaded three-bar planar truss and its tip deflection (δ) subjected to stress constraint on each truss member. The schematic is shown in Figure 3 and the mathematical statement of this optimization problem (Haftka et al., 1993) is written as follows :

$$\text{Minimize} \quad W(A_1,A_2)\ \&\ \delta(A_1,A_2) \tag{5}$$

$$\text{subject to} \quad \sigma_1(A_1,A_2)\leq\sigma^{upper}=20 \tag{6}$$

$$\sigma_2(A_1,A_2)\leq\sigma^{upper}=20$$

$$\sigma_3(A_1,A_2)\leq\sigma^{lower}=-15$$

$$0.1\leq A_1,A_2\leq 5.0$$

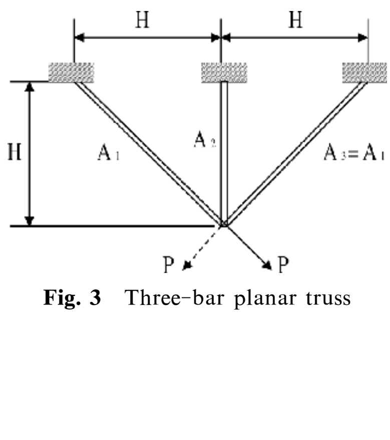


Fig. 3 Three-bar planar truss

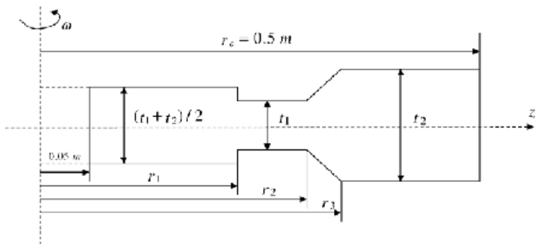


Fig. 4 Flywheel

Table 1 Problem parameters in flywheel design

parameter	value
Yield stress, σ_Y	1.48E9 N/m^2
Rotating speed, ω	2000 rad/sec
Poisson ratio, ν	0.3
Material density, ρ	7830 kg/m^3
Initial weight, W_o	2171.35 kg

3.2 Flywheel

For a flywheel design as shown in Figure 4, the objective is to determine radius and thickness variables by maximizing the kinetic energy (KE) and minimizing the disk weight (W) subjected to weight and yield stresses. The optimization problem (Mistree et al., 1994) is stated as follows:

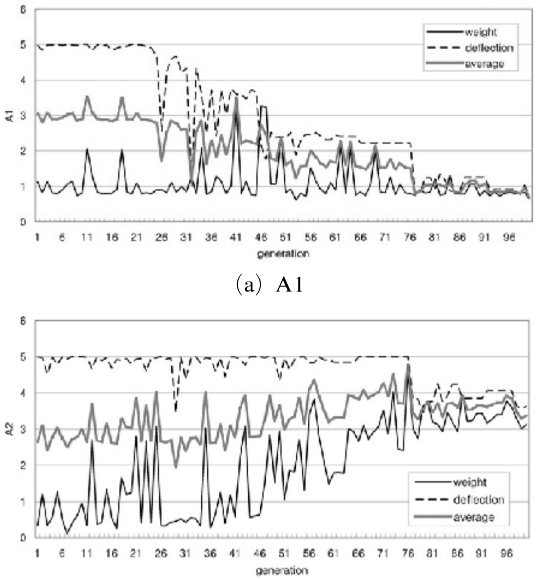
$$\text{Minimize} \quad \frac{1}{KE(r_1, r_2, r_3, t_1)} \;\&\; W(r_1, r_2, r_3, t_1) \tag{7}$$

$$\begin{aligned} \text{subject to} \quad & \sigma_R \leq \sigma_Y \\ & \sigma_T \leq \sigma_Y \\ & t_1 \leq t_2 \;\&\; 0.01 \leq t_1 \leq 0.1 \\ & 0.05 \leq r_1, r_2, r_3 \leq 0.5 \qquad (\text{unit: m}). \end{aligned} \tag{8}$$

In the above problem, σ_R and σ_T are denoted as radial and tangential stresses, respectively, and are limited by the yield stress, σ_Y. Problem parameters including material properties are summarized in Table 1.

4. Results and Discussion

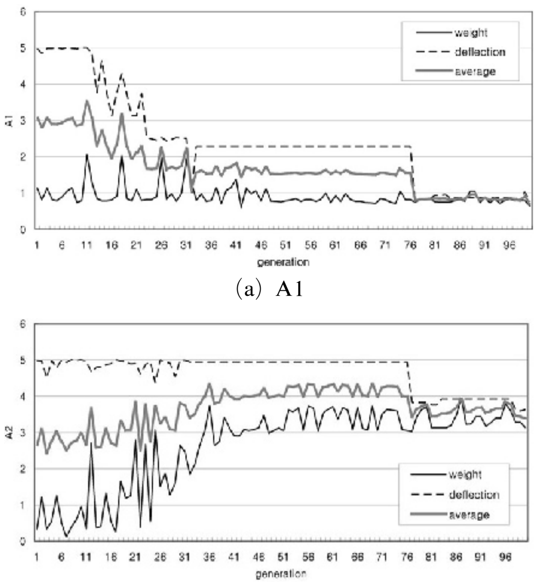
Coevolution based multiobjective optimization is conducted using prescribed scheduling schemes in Figure 2. Typical results are shown in Figures 5 and 6, wherein optimized cross sectional areas of three-bar truss are presented. Since POD values are reduced to 0.1 in this case, optimal solutions are obtained when the overall difference between design variables for two objective functions is less than 0.1. As mentioned before, this corresponds to a case where a weighting factor is



(a) A1

(b) A2

Fig. 5 Three-bar truss results by sine wave type POD



(a) A1

(b) A2

Fig. 6 Three-bar truss results by half sine wave type POD

0.5 used in the context of weighted method. It is detected that solution histories in Figure 5 is more fluctuating than those in Figure 6. The faster scheduling with half sine wave type is shown to be more efficient in this optimization problem. For the verification of adaptive POD scheme is explored to see how such approach affect the design results. Based on Eq. (4), CEC is conducted in three-bar truss problem as well. The resulted APOD history is shown in Figure 7, and its corresponding optimized design solutions are demonstrated in Figure 8. It is noted that the resulted APOD behavior is more similar to half sine wave rather than sine wave so that solution histories in Figure 8 are also similar to those in Figure 6. APOD would be noticeable when a careful selection of prescribed scheduling in POD is not available. Coevolution results with other scheduling schemes are summarized in terms of optimized design solutions and NCALL, the number of function calls as shown in Table 2. APOD is a compromise between sine wave type POD and half sine wave type POD in terms of NCALL.

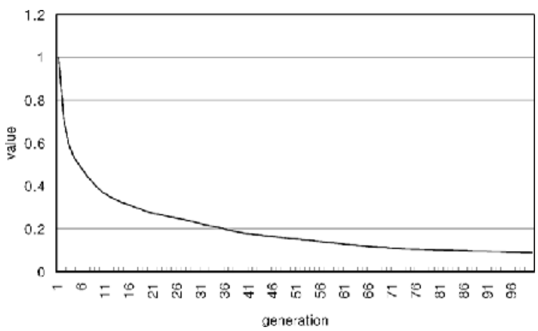
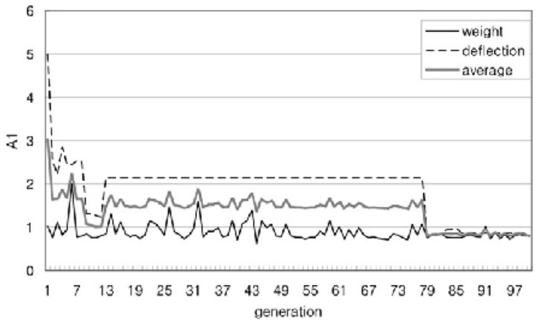
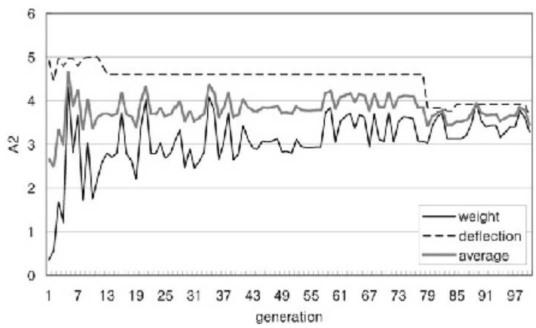


Fig. 7 Resulted APOD with k=0.5 in three-bar truss design

Above results are obtained by assigning POD_{final}=0.1. Now, it is necessary to identify Pareto optimal solution by controlling the final value of POD. Multiobjective Pareto solutions in the present study can be produced by changing POD_{final} ranging from 0.1 to 0.9. It is reminded that POD_{final}=0.1 is a process for commonly compromising between two objective function values while POD_{final}=0.9 induces more consideration of single objective function values. Pareto solutions obtained from the present approaches and a traditional weighted method by Eq. (3) are shown in Figure 9,



(a) A1



(b) A2

Fig. 8 Three-bar truss results by APOD with k=0.5

Table 2 Three-bar truss design results

POD	A_1	A_2	F_1 (W)	F_2 (δ)	NCALL
Linear ramp	0.7100	3.3800	4.9685	3.3878	6240
Sine wave	0.7100	3.3800	4.9685	3.3878	6060
Half linear ramp	0.7100	3.3800	4.9685	3.3878	4660
Half sine wave	0.7100	3.3800	4.9685	3.3878	4620
APOD (k=0.5)	0.7900	3.3455	5.5427	3.3820	4740

NCALL : the total number of function calls

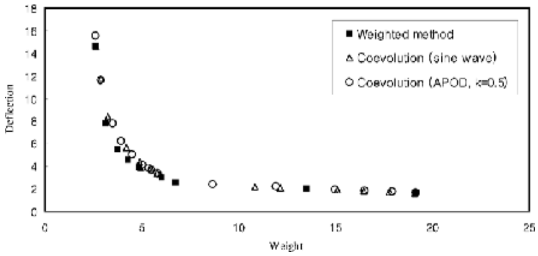


Fig. 9 Pareto optimal solutions in three-bar truss design

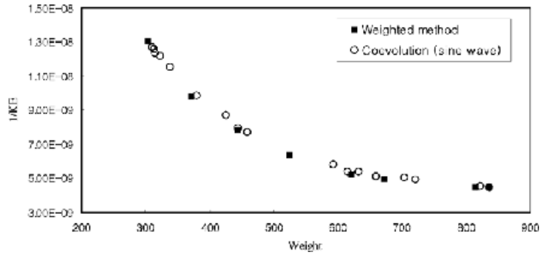


Fig. 10 Pareto-optimal solutions in flywheel design

Table 3 Multiobjective optimization of flywheel

(a) weight optimization by coevolution

POD_{final}	r_1	r_2	r_3	t_1	weight	$1/KE$	KE
0.1	0.06	0.319	0.341	0.05	444.63	7.899E−09	1.266E+08
0.2	0.125	0.283	0.298	0.051	425.774	8.665E−09	1.154E+08
0.3	0.182	0.364	0.375	0.052	379.953	9.830E−09	1.017E+08
0.4	0.182	0.271	0.273	0.053	339.632	1.155E−08	8.658E+07
0.5	0.249	0.266	0.268	0.05	322.884	1.218E−08	8.212E+07
0.6	0.249	0.266	0.268	0.05	322.884	1.218E−08	8.212E+07
0.7	0.059	0.346	0.348	0.05	315.948	1.233E−08	8.112E+07
0.8	0.118	0.212	0.213	0.05	313.734	1.257E−08	7.954E+07
0.9	0.183	0.353	0.354	0.05	310.636	1.266E−08	7.897E+07

(b) $1/KE$ optimization by coevolution

POD_{final}	r_1	r_2	r_3	t_1	weight	$1/KE$	KE
0.1	0.093	0.311	0.334	0.05	458.6	7.694E−09	1.300E+13
0.2	0.091	0.295	0.332	0.055	593.9129	5.811E−09	1.721E+08
0.3	0.092	0.301	0.345	0.055	632.743	5.362E−09	1.865E+08
0.4	0.063	0.306	0.354	0.05	615.85	5.372E−09	1.862E+08
0.5	0.051	0.281	0.33	0.051	660.04	5.091E−09	1.964E+08
0.6	0.145	0.266	0.31	0.058	703.906	5.029E−09	1.988E+08
0.7	0.152	0.256	0.303	0.055	721.941	4.917E−09	2.034E+08
0.8	0.189	0.21	0.264	0.051	822.117	4.497E−09	2.224E+08
0.9	0.19	0.196	0.25	0.051	837.365	4.466E−09	2.239E+08

(c) weight method

ω_1	r_1	r_2	r_3	t_1	weight	$1/KE$	KE
0	0.19	0.196	0.25	0.051	837.365	4.466E−09	2.239E+08
0.1	0.054	0.182	0.237	0.05	814.359	4.450E−09	2.247E+08
0.2	0.064	0.296	0.35	0.051	672.860	4.910E−09	2.037E+08
0.3	0.052	0.295	0.35	0.05	672.747	4.888E−09	2.046E+08
0.4	0.086	0.327	0.381	0.05	621.872	5.226E−09	1.913E+08
0.5	0.056	0.327	0.364	0.05	525.356	6.362E−09	1.572E+08
0.6	0.054	0.335	0.359	0.05	445.138	7.784E−09	1.285E+08
0.7	0.085	0.363	0.376	0.05	371.980	9.770E−09	1.024E+08
0.8	0.241	0.346	0.346	0.05	304.408	1.301E−08	7.686E+07
0.9	0.201	0.345	0.345	0.05	304.408	1.301E−08	7.686E+07
1	0.167	0.346	0.346	0.05	304.408	1.301E−08	7.686E+07

wherein there demonstrates the good agreement among each of design methods. Another design problem of a rotating disk in flywheel is explored to generated Pareto optimal solutions as well. In the coevolution strategy, a sine wave type POD is selected because it shows a marginal performance in the three-bar truss problem. Results from coevolution and a weighted method are presented in Figure 10, and their numerical data is summarized in Table 3. These also show similar trends to a three-bar truss problem.

5. Closing Remarks

The concept of cooperative coevolution is employed in the context of GA based multiobjective optimization. Coevolutionary optimization is a parallel process to obtain optimized design solutions by reflecting others' environments. In the present study, two approaches of scheduling schemes for considering such reflection are implemented. Premature solutions get more compromise as the genetic coevolution is in progress. There shows a good agreement between proposed strategies and a traditional weighed method in multiobjective optimization. For further study, both competitive coevolution and host-parasites methods are being considered to enhance the design performance in the context of multiobjective optimization using GA based CEC.

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